Curriculum RP-12

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DEPARTMENT OF EDUCATION

MATHEMATICS

THE FOUR- AND FIVE-YEAR PROGRAMMES, ALL BRANCHES GRADE 9

Observations on the Course of Study, to be Read with Curriculum I. 1, 1951 Revised.

THE FIVE-YEAR PROGRAMME, ALL BRANCHES GRADES 10, 11 AND 12

Observations on the Courses of Study to be Read with Curriculum I. 1, 1951 Revised and Curriculum S. 12, 1961.

THE FOUR-YEAR PROGRAMME, GRADES 10, 11 AND 12

Curriculum I. and S. 27 to Remain in Effect for The Science, Technology and Trades Branch.

Comments on Curriculum I. and S. 8 which Remains in Effect for the Business and Commerce Branch.

New Courses of Study for the Arts and Science Branch for Introduction in Grade 10 in September, 1963.

THE TWO-YEAR PROGRAMME

These courses are experimental in that they will be subject to review. Suggestions for their improvement will be welcomed.

MATHEMATICS

for

Grade 9

in

The Four - and Five-Year Programmes, All Branches

A. General Observations

1. Curriculum I:1 of 1951 (pages 157 to 165) contains suggestions and course details for mathematics which have applied for all pupils of Grade 9. It is to be noted that the suggestions and course details outlined below differ from those of Curriculum I:1 only in those features suggested by teacher experience over the last ten years and by the fact that this course is planned for pupils whose placement indicates intent of spending at least four years in secondary school.

   Referring specifically to pages 157 to 159, the general observations there developed might be considered as applicable with the following restrictions and emphasis:

   (a) Since this course is restricted to those whose placement indicates intent of spending at least four years in secondary school, one can be somewhat more definite regarding suggested content and variations than is indicated in general observation "1" on pages 157 and 158. It will be noted that the detail below actually suggests which sections should be taken by those who have chosen a four-year programme at the start of Grade 9 and by those who may make the choice of a four-year or five-year programme after Grade 9.

   (b) General observations "2" and "3" on page 158 suggest the desirability of correlation of the various branches of mathematics. Teacher experience over the last few years has indicated the desirability of using a definite order of topics as an aid in achieving this aim. This order is indicated in the course detail below.

   (c) The suggestions on the whole of page 159 covering integrated practice on fundamental operations, the importance of the oral approach to mathematics, and the importance of stress in mathematics of proper oral and written English must be considered as vital parts of the suggestions for mathematics.

2. Sections for each group

   (a) Those pupils who have chosen a four-year programme at the start of Grade 9 should cover at least the following sections: 1, 2, 3, 4, 5, 6, and either 8 or 9.

   (b) Those pupils whose placement in either the four-year or five-year programme is to come after Grade 9 should cover at least sections 1 to 8 inclusive.

3. Where it is desired to use the number structure approach, the teacher after consultation with the principal, will find it necessary to add the following items to the course at the appropriate places: the concept of set, subset, and null set; the extension of solutions and graphing to include inequalities; set-builder notation; the recognition of the commutative, associative, and distributive properties and their use in the development of the operations; and the classification of number into the natural numbers and zero, the integers, the rationals, and the reals.

B. Course Details

Section 1. The Concept of Variable

Evaluation of simple algebraic expressions in one variable using zero and natural number values only.

Intuitive solution of simple equations of two or three terms with one term involving a variable.

Study of simple algebraic expressions to develop the concepts of term, factor, monomial, binomial, trinomial, coefficient.

Powers and indices with evaluations of powers involving one variable.

Order of Operations.

Evaluation of algebraic expressions involving more than one term and more than one variable.

Section 2. Operations with Signed Numbers

Need of 'Signed Numbers'.

Occurrence of 'Signed Numbers'.

Addition and Subtraction of 'Signed Numbers'.

Review of multiplication and division of (arithmetical) fractions and decimals.

Multiplication and Division of positive monomials involving fractional and decimal coefficients.

Multiplication and Division of monomials involving powers.

Multiplication and Division of 'Signed Number' monomials.

Multiplication of monomials and binomials.

Multiplication of two binomials.

Simplifying of algebraic expressions involving these multiplications (rules for removing brackets treated as multiplication by + 1 or - 1)

Section 3. Solution of Equations

Review of the intuitive solution of equations suggested in section 1 above, and extension of this method to more difficult examples which may need reduction first to one of the following three forms $2x + 5 = 15$, $13 = 4x - 5$, $12x - 3 = 2x + 9$.

Solution of equations using the axioms.

Extension to solution of more difficult cases.

Graphing of solutions on the number line.

Section 4. Problems solved by Equations

Practice on conversions of given sequences of operations to algebraic form.

Practice on conversion of given relations to equations.

Algebraic solution of problems involving number, ages, and amounts of money.

Review of calculations of percentage and rate per cent.

Solution by equations of problems involving percentage.

Section 5. Mensuration

Review and extension of concepts of the set of straight-line simple closed figures to include triangle, quadrilateral, polygons.

Review and extension of the set of triangles to include equilateral, isosceles, and scalene.

Review and extension of the set of quadrilaterals to include rectangle, square, parallelogram, trapezium, rhombus.

Review of definitions of the circle.

Review concept of perimeter and application to the straight line closed figures.

Development of the formulas for: areas of the quadrilaterals, area of a triangle ($\frac{1}{2}$ bh), perimeter of a circle, and area of the region bounded by the circle. Calculations based on these formulas stressing formula manipulation by use of the axioms (manipulation where necessary before substitution)

Algebraic solutions of problems involving measurements

Section 6. Arithmetic Factoring and Operations with Fractional Expressions.

Review of factor, prime number.

HCF and LCM by the factor method.

Review addition and subtraction of fractions and decimals

Simplification of complex fractions (stressing the use of the principle of equivalent fractions)

Square root by the factor method and by estimation

Square root by the formal method (after considerable practice with the above-mentioned methods)

Section 7. Algebraic Factoring and Operations with Fractional Expressions.

Review of Special Products of Monomials and Binomials

Common Factor

Factoring of Trinomials

Factoring Difference of Squares

Reduction of Fractions to Lowest Terms

Multiplication of Fractional Expressions

Division of Fractional Expressions

HCF and LCM

Addition and subtraction of Fractional Expressions

Equations extended to include these operations.

Section 8. Geometry

Inductive approach to the various properties and relations listed in Curriculum I:1 (page 160).

Development of the geometrical language of construction descriptions by the use of the six fundamental constructions.

Section 9. 'Social' Graphs and Business Arithmetic

Interpretation of Graphs

The topics of business Arithmetic as outlined in Curriculum I:1 (pages 163 and 164).

MATHEMATICS

Grades 10, 11, and 12, Five-Year Programmes
in
All Branches

General Observations

1. Except for the fact that mathematics in Grade 10 is now compulsory in this programme the first general observation of page 165 of Curriculum I:1 may be considered as applicable for all three grades 10, 11, and 12 covered by this bulletin. Furthermore, all pupils taking the mathematics of the five-year programme are by that choice of programme indicating the intention of proceeding through Grade 13 to further studies. In the main, pupils taking Mathematics in these grades must be considered as doing so with the intention of eventually taking one or more of the three grade thirteen mathematics units.

2. Some of the changes being discussed for Grade 13 will likely have an effect on the allotment of topics in Grade 10, 11, and 12. In view of this possibility of change within the next few years, it is suggested that, for the time being, the courses for these grades remain as indicated in Curriculum I:1 (Course 1 pages 166 and 167) and as specified in Curriculum S.12 (1961 re-printing). It is expected, however, that the following suggestions will be found useful:

   (a) The amount of time spent in Grade 10 on review of Grade 9 algebra and geometry may be reduced to allow completion of the algebra section before Christmas and the geometry section earlier in the second term so that the section on application of the Pythagorean relation to mensuration of solids may be completed before the end of the year.

   (b) The amount of time spent in Grade 11 on the review of Book I may be reduced if it has been properly covered in Grade 10 as a result of the above suggestions or as a result of teaching time beyond the minimum specifications. This reduction of time on Book I in Grade 11 should make it possible to take advantage of the coverage of additional topics as suggested in note 8 on page 10 of Curriculum S. 12. The "additional mathematics topics" are not specified there, but it is generally accepted that these would be such as indices and logarithms, and the introduction to trigonometry, and possibly, although not so generally accepted, an introduction to analytic geometry.

   (c) In any case, it is strongly urged that the graphical approach to algebraic topics be stressed at all stages from the number-line graphing of Grade 9, to the stage reached in this approach as indicated in section 7 on page 17 of Curriculum S. 12.

3. It is to be noted that students of the Science, Technology, and Trades Branch follow the same course as the other two branches. This agrees with the time-tested practice followed for many years in those technical schools having a General-Technical Branch leading to Grade 13. It is accepted, however, that teachers will use applications to and from drafting and shop work where possible. It is also accepted as good practice that the two Grade 11 'optional' topics of logarithms and trigonometry are covered early in Grade 11 to prepare the pupils for the use of the slide rule and the solution of right-angled triangles in the shop work.

4. It is to be noted that students of the Business and Commerce Branch also follow the same course. Every student in this Branch should also have some practice in the mathematics of business and commerce. Because every such student should have this practice and some would not get the opportunity if such work were included in the mathematics option, the decision was made that this section should be a part of the work covered in the compulsory 'Commercial' time, probably related to Accountancy and Business Machines.

5. The mathematics teacher's responsibility for the quality of the pupils' oral and written English as outlined in note 5 of Curriculum I:1 (page 159) should be borne in mind constantly.

MATHEMATICS

Grades 10, 11, and 12, Four-Year Programme in All Branches

A. Science, Technology, and Trades

Courses for this branch as outlined on pages 18 to 27 of Curriculum I and S. 27 have been found to be very satisfactory and it is suggested that these be continued with proper attention to the general observations and aims as outlined on pages 14 and 15, and with due regard to need of the teacher assuming responsibility for the quality of pupils' oral and written English as suggested in note 5 on page 159 of Curriculum I:1.

B. Business and Commerce

This course is intended to suit the needs of those pupils who do not desire to prepare themselves for the mathematics of Grade 13 but who wish further mathematical training in preparation for business life. These pupils intend to obtain a Secondary School Graduation Diploma in the four-year Business and Commerce programme.

It is suggested that Courses I, II, and III as outlined on pages 14 and 15 of Curriculum I and S. 8 be considered the outline of the courses for Grades 10, 11, and 12, respectively.

The suggestions given on page 14 may be considered as applicable except the references to the coverage of Course I in Grade 9. Some pupils who were enrolled in a Business and Commerce four-year programme in their grade nine year will have touched upon some applications of percentage in section 9 of the grade nine course but it will not have been a complete coverage of Course I. In any case, there will likely be some pupils in every grade ten class who will not have covered section 9 in grade nine. Hence, the planning for grade ten should be on the basis that Course I will not have been covered in Grade 9.

In addition to those on page 14, it is thought that the following suggestions should be given particular attention.

1. Pupils should be given more practice in estimating answers.

2. The growth of automation in business seems to indicate that more consideration must be given to the use of formulas. This, among other reasons, seems to point toward the need of more practice in the manipulation of and substitution in formulas. It is therefore suggested that a unit on formula manipulation and evaluation be added to Course I and that this skill be maintained throughout by practice not only with business arithmetic formulas, but also with those from mensuration and science. As formulas are really algebraic equations and should be treated as such, the meaningful approach to formula manipulation requires the knowledge of algebra up through the operations with monomials and simple equations such as is provided in sections 1, 2, 3, and 4 of the Grade 9 course.

C. Arts and Science

General Observations

1. This course is intended to suit the requirements of those pupils who have no need to prepare themselves for the mathematics of Grade 13 and are not taking the somewhat skill-slanted mathematics of the Business and Commerce or Science, Technology, and Trades branches. It therefore attempts to provide a 'mathematics-for-life' with a general, rather than special, mathematical approach.

2. The courses for Grades 10 and 11 are planned to contain obligatory topics only and are aimed to reach a minimum level in the mathe-tics-for-life. The Grade 12 course as outlined should be considered to be more flexible according to need and should be adjusted to fit the local situation.

3. It should be noted that the Grade 11 course is a prerequisite for the Grade 12 course and that a pupil in Grade 12 who wished mathematics and who had dropped this option after Grade 10 would have to take the Grade 11 course.

4. It might be noted also that transfer into this Grade 11 course would be possible from the end of Grade 10 of any of the branches of the five-year programme provided that the introduction to the Grade 11 trigonometry be such as to cover a complete teaching of the Grade 10 unit in trigonometry.

5. It is hoped that the attempt to incorporate arithmetic into the algebra, and all phases of mathematics into a whole, will prove useful.

6. It is being realized more and more that pupils should be encouraged to approach mathematics problems with an intuitive outlook. This is particularly fruitful in problems of locus and geometrical con-figurations. It should also be encouraged in calculation problems by such devices as estimating answers and checking the reasonable-ness of an answer.

Details of the Course

Grade 10

Section 1. Algebra and Applications

Review addition, subtraction, multiplication, and division of rational expressions (including indicated operations)

Extension of solutions of equations in one variable to in-clude more difficult fraction types

Problems solved by equations in one variable

Type products developed as patterns

Factoring of common factor, trinomial, difference of two squares

Extension of trinomial factoring to more difficult types

Numerical evaluations of algebraic expressions

Manipulation of formulas

Review of application of linear and area formulas (rectangle, triangle, parallelogram, trapezium, circle, hexagon, equilateral triangle)

Extension of mensuration to include the applications of the Pythagorean Theorem and the lateral surfaces and volumes of (a) Cylinder and Prism (b) Cone and Pyramid and (c) Sphere (Problems on volume of sphere restricted to those not requiring formal cube root or logarithms)

Graphing of solution sets in one variable (including inequalities)

Graphing extended to two variables stated in formulae

Cartesian co-ordinate system

Graphing of linear algebraic expressions

Graphical solution of simultaneous linear equations

Algebraic solution of simultaneous linear equations

Section 2. Applications of Percentage

Review of percentage and rate per cent

Applications to Profit and Loss, Trade Discounts, Marketing, Retail Tax, Take-Home Pay, Selling and Buying Commissions, Simple Interest, Savings Account Interest, Canada Savings Bond Interest

Section 3. Geometry

Review of definitions relating to angles and triangles

Measurement and construction of angles and triangles

Review of the six fundamental constructions of grade nine

Inductive study of geometrical properties - vertically opposite angles, congruencies, isosceles and equilateral triangles, parallel lines, parallelograms, exterior angle, similar triangles

Review of circle definitions. Extend to include tangent and secant

Inductive discovery of angle properties in circle and semi- circle

Construction of regular polygons in circles, circumscribed and inscribed circles, tangents from points on or outside the circle

Section 4. Trigonometry

Trigonometric functions of angles

Ratios for acute angles using terms opposite, hypotenuse, and adjacent. Reciprocal and Pythagorean relations

Ratios by measurement and calculation

Trigonometric ratios of 45^{o} , 30^{o} , 60^{o} , and evaluations based thereon

Use of the trigonometric tables for calculation of unknown distances and angles.

Grade 11

Section 1. Algebra

Review and extension to more difficult examples of fundamental operations with rational algebraic expressions

Review and extension to more difficult examples of graphical and algebraic solutions of linear equations of two variables

Review of manipulation with indices and extension to concepts of zero, negative, and fractional indices

Calculations of multiplication and division applying the standard number method

Logarithms for multiplication, division, roots and powers

Slide Rule

Review of manipulation of formulae with applications to those of mensuration and other fields

Extension of mensuration to cover segments, frustums, hollow solids, sphere

Review of factoring the common factor, difference of squares, and trinomial and extension to sum and difference of cubes developed through the factor theorem

Ratio and Proportion applications to arithmetic, physics, development of mean proportional, continued proportion, variation

Quadratic Surds - addition, subtraction, multiplication, division

Section 2. Geometry

Review of the definitions and properties developed in Grade 10

Inductive development of properties of similar triangles - the parallel - proportion property, perpendicular to hypotenuse property, side proportion by bisector of angle. Application to constructions

Section 3. Trigonometry

Review of concepts and applications from Grade 10

Definitions of trigonometric functions by cartesian co-ordinate system

Use of unit circle to show variations from 0° to 90°

Extension to angles of the second, third and fourth quadrants

Development of the law of sines (acute-angled triangle only)

Demonstration of development of the cosine law

Application of sine law and cosine law in the solution of triangles

Section 4. Applications of Percentage

Review from Grade 10 with more difficult examples of profit and loss, discounts, commission, simple interest

Calculation of Home Owner Costs - taxes, insurance, comparisons with rental

Calculation of Car Owner Costs - license, car insurance, upkeep

Calculations related to banking and personal or bank loans; interest on and discounting of promissory notes

Compound interest calculation by using the cumulative interest method rather than the formula method; determination of the effective rate in various cases of time payments. Calculation of costs of various security investments - life insurance, accident and health insurance, pensions

Section 5. Introduction to Common Statistical Terms

Concepts and simple applications of mean, median, range, decile, quartile, and percentile calculated without interval grouping.

Grade 12

Section 1. Algebra

Review and extension to more difficult examples of the Grade 11 work in fundamental operations and equations

Extension of solution of linear equations to three variables

Quadratic equations in one variable - solution by factoring, formula (proof demonstrated but not required to be developed by pupils), graphical (include recognition of imaginary roots)

Quadratic equations in two variables - graphical and algebraic solutions (Simpler types only, e.g. line-circle and line-parabola)

Series developed as patterns, arithmetic and geometric, calculation of means, simpler cases of determination of missing terms.

Sums of series - to n terms developed with practice on formulas

Section 2. Mensuration and Trigonometry

Review of the manipulation of formulas and extension to more difficult formulas from mensuration and other fields

Evaluation from formulas (using logarithms and/or slide rule)

Review of the trigonometric functions and extension to more difficult examples of calculation of missing angles or lengths in right-angled triangles

Introduction to the use of the logarithms of the trigonometric functions

Development of the sine law for acute-angled triangles (only), and applications thereon

Development of the sine formula for the area of a triangle, and applications thereon

Demonstration of the algebraic proof of the s formula for the area of a triangle, and applications thereon

Section 3. Applications of Percentage

Extension of compound interest problems to include development of the formula and the use of tables

Review of the calculation of home owner costs and extensions to include home purchase costs such as realtor fees and mortgages. Calculations of effective rate on various time and rate arrangements for first and second mortgages.

Review of the calculations of car owner costs stressing the effective rate of interest on time payments. Extension of these calculations to general instalment buying.

Problems related to the operation of a business - calculation of 'overhead', determination of proper mark-up and margin, calculation of possible discounts, calculations related to the shipping of goods and the transmission of payment, calculations of best-buys, determination of net profits and proper distribution in a partnership or corporation. Simple problems related to the raising of additional capital.

Calculations related to financing by elected bodies - various types of taxes (customs, sales, property, income); problems of distribution among the various government levels - (provincial tax agreements, provincial government grants, county taxation); arrangements for borrowing (bonds, debentures, sinking funds).

Review from Grade 11 calculations of the costs of various security investments and the extension to include calculations of cost of purchase of stocks, bonds, debentures, or sinking funds. Simple problems of effective yield on purchase of bonds or debentures (using Makeham's formula). Calculations of the indicated yield on the purchase of dividend-paying stock.

Section 4. Geometry

Review of definitions and properties developed in Grades 10 and 11. Various constructions involving applications of these properties and the synthetic concept of locus.

Review of the co-ordinate system used in the algebra and trigonometry.

Problems on area and distances on geometric figures obtained by plotting of points.

Development and practice in the use of the formula for the distance between two points.

Co-ordinates of the mid-point of a straight line.

Development of the concept of slope of line.

Use of the conditions for parallelism and perpendicularity of lines.

Development of the point-slope and of the two-point equations of the straight line.

(Optional Topic)

Approximately ten to twelve theorems developed deductively as examples of proof to include applications of congruency, inequality, superposition, converse, and indirect proofs. Include examples of deductive proofs from algebra, from legal field, from investigation field, from advertisements, and from public speaking.

Grade Nine and Ten Mathematics

(Two-Year Programme)

It is generally conceded that all pupils undertaking this programme will require more and continuing practice in manipulation of fractions and decimals and in their applications to measurement and percentage.

At the same time there seems to be some agreement that any pupil who completes this two-year programme successfully might be allowed to transfer to the beginning of Grade 10 of one of the branches of a four-year programme. This would require that these pupils have covered in the two years the same algebra and geometry as that covered in one year by the pupils in Grade 9 who have chosen a four-year programme at the start of Grade 9. Even without this preparation for possible transfer there is some considerable support for the device of using the new field of algebra for part of the great amount of practice needed in arithmetic manipulation, and certainly the required applications to measurement would support sufficient algebra for the manipulation of formulas. The extension of the knowledge of geometrical figures beyond that covered in Grade 8 (see Curriculum I:1 page 156) to what is ordinarily covered in Grade 9 needs no defence.

The suggestion for courses for these pupils is therefore that <u>in the two years</u> these pupils cover as a minimum the sections 1, 2, 3, 4, 5, 6, 8 <u>and</u> 9 of the four - and five-year Grade 9 course and supplement this with expanded units of review and practice at arithmetic manipulations at strategic points. It is taken for granted that development of new concepts will be supported by a beyond-the-ordinary number of oral and written easier examples.

Grade Nine and Ten Mathematics

(Occupational Programme)

The course in mathematics for this programme will be found in <u>Suggested Courses for the Diversified Occupational Programme.</u>

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